Indices And Cube Root

Practice set 3.1

Q. 1. Express the following numbers in index form.

(1) Fifth root of 13
(2) Sixth root of 9
(3) Square root of 256
(4) Cube root of 17
(5) Eighth root of 100
(6) Seventh root of 30

Answer : (1) Fifth root of 13

In general, n^{th} root of 'a' is expressed as $a^{\frac{1}{n}}$.

So, the fifth root of 13 is expressed as $13^{\frac{1}{5}}$.

Here, 13 is base, $\frac{1}{5}$ is the index and $13^{\frac{1}{5}}$ is the index form of the number.

(2) Sixth root of 9

In general, n^{th} root of 'a' is expressed as $a^{\frac{1}{n}}$.

So, the sixth root of 9 is expressed as $9^{\frac{1}{6}}$.

Here, 9 is base, $\frac{1}{6}$ is the index and $9^{\frac{1}{6}}$ is the index form of the number.

(3) Square root of 256

In general, n^{th} root of 'a' is expressed as $a^{\frac{1}{n}}$.

So, the square root of 256 is expressed as $256^{\frac{1}{2}}$.

Here, 256 is base, $\frac{1}{2}$ is the index and $256^{\frac{1}{2}}$ is the index form of the number.

(4) Cube root of 17

In general, n^{th} root of 'a' is expressed as $a^{\frac{1}{n}}$.

So, cube root of 17 is expressed as $17^{\frac{1}{3}}$.

Here, 17 is base, $\frac{1}{3}$ is the index and $17^{\frac{1}{3}}$ is the index form of the number.

(5) Eighth root of 100

In general, n^{th} root of 'a' is expressed as $a^{\frac{1}{n}}$.

So, the eighth root of 100 is expressed as $100^{\frac{1}{8}}$.

Here, 100 is base, $\frac{1}{8}$ is the index and $100^{\frac{1}{8}}$ is the index form of the number.

(6) Seventh root of 30

In general, n^{th} root of 'a' is expressed as $a^{\frac{1}{n}}$.

So, the seventh root of 30 is expressed as $30^{\frac{1}{7}}$.

Here, 30 is base, $\frac{1}{7}$ is the index and $30^{\frac{1}{7}}$ is the index form of the number.

Q. 2. Write in the form 'n^{th} root of a' in each of the following numbers.

1. $(81)^{1/4}$ 2. $(49)^{1/2}$
3. $(15)^{1/5}$ 4. $(512)^{1/9}$
5. $(100)^{1/19}$ 6. $(6)^{1/7}$

Answer : 1. $(81)^{1/4}$

In general, $a^{1/n}$ is written as 'n^{th} root of a'.

So, $(81)^{1/4}$ is written as '4^{th} root of 81'.

2. $(49)^{1/2}$

In general, $a^{1/n}$ is written as 'n^{th} root of a'.

So, $(49)^{1/2}$ is written as 'square root of 49'.

3. $(15)^{1/5}$

In general, $a^{1/n}$ is written as 'n^{th} root of a'.

So, $(15)^{1/5}$ is written as '5^{th} root of 15'.

4. $(512)^{1/9}$

In general, $a^{1/n}$ is written as 'n^{th} root of a'.

So, $(512)^{1/9}$ is written as '9^{th} root of 512'.

5. $(100)^{1/19}$

In general, $a^{1/n}$ is written as 'n^{th} root of a'.

So, $(100)^{1/19}$ is written as '19^{th} root of 100'.

6. $(6)^{1/7}$

In general, $a^{1/n}$ is written as 'n^{th} root of a'.

So, $(6)^{1/7}$ is written as '7^{th} root of 6'.

Practice set 3.2

Q. 1. Complete the following table.

Sr. No.	Numbers	Power of the root	Root of the power
(1)	$(225)^{3/2}$	Cube of square root of 225	Square root of cube of 225
(2)	$(45)^{4/5}$		
(3)	$(81)^{6/7}$		
(4)	$(100)^{4/10}$		
(5)	$(21)^{3/7}$		

Answer :

Sr. No.	Numbers	Power of the root	Root of the power
(1)	$(225)^{3/2}$	Cube of square root of 225	Square root of cube of 225
(2)	$(45)^{4/5}$	Fourth power of fifth root of 45	Fifth root of fourth power of 45
(3)	$(81)^{6/7}$	Sixth power of seventh root of 81	Seventh root of sixth power of 81
(4)	$(100)^{4/10}$	Fourth power of tenth root of 100	Tenth root of fourth power of 100
(5)	$(21)^{3/7}$	Cube of seventh root of 21	Seventh root of cube of 21

Explanation of Table

Generally we can express two meaning of the number $a^{m/n}$.

$a^{m/n} = (a^m)^{1/n}$ means 'n^{th} root of m^{th} power of a'.

$a^{m/n} = (a^{\frac{1}{n}})^m$ means 'm^{th} power of n^{th} root of a'.

(1) $(225)^{3/2}$

$(225^3)^{1/2}$ means 'Cube of square root of 225'.

$(225^{1/2})^3$ means 'Square root of cube of 225'.

(2) $(45)^{4/5}$

$(45^4)^{1/5}$ means 'Fourth power of fifth root of 45'.

$(45^{1/5})^4$ means 'Fifth root of fourth power of 45'.

(3) $(81)^{6/7}$

$(81^6)^{1/7}$ means 'Sixth power of seventh root of 81'.

$(81^{1/7})^6$ means 'Seventh root of sixth power of 81'.

(4) $(100)^{4/10}$

$(100^4)^{1/10}$ means 'Fourth power of tenth root of 100'.

$(100^{1/10})^4$ means 'Tenth root of fourth power of 100'.

(5) $(21)^{3/7}$

$(21^3)^{1/7}$ means 'Cube of seventh root of 21'.

$(21^{\frac{1}{7}})^3$ means 'Seventh root of cube of 21'.

Q. 2. Write the following number in the form of rational indices.

(1) Square root of 5th power of 121.
(2) Cube of 4th root of 324.
(3) 5th root of square of 264.
(4) Cube of cube root of 3.

Answer : We know that 'nth root of mth power of a' is expressed as $(a^m)^{1/n}$

And 'mth power of nth root of a' is expressed as $(a^{\frac{1}{n}})^m$.

(1) Square root of 5th power of 121.

We know that,

'nth root of mth power of a' is expressed as $(a^m)^{1/n}$

So, 'Square root of 5th power of 121' is expressed as $(121^5)^{1/2}$ or $(121)^{5/2}$.

(2) Cube of 4th root of 324.

We know that,

'nth root of mth power of a' is expressed as $(a^m)^{1/n}$

So, 'Cube of 4th root of 324' is written as $(324^{1/4})^3$ or $(324)^{3/4}$.

(3) 5th root of square of 264.

We know that,

'nth root of mth power of a' is expressed as $(a^m)^{1/n}$

So, '5th root of square of 264' is written as $(264^2)^{1/5}$ or

$(264)^{2/5}$.

(4) Cube of cube root of 3.

We know that,

'm^{th} power of n^{th} root of a' is expressed as $(a^{\frac{1}{n}})^m$

So, 'Cube of cube root of 3' is written as $(3^{1/3})^3$ or $(31)^{3/3}$.

Practice set 3.3

Q. 1 A. Find the cube root of the following numbers.

8000

Answer : First find the factor of 8000

$8000 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5$

For finding the cube root, we pair the prime factors in 3's.

$= (2 \times 2 \times 5)^3$

$= (2 \times 10)^3$

$= 20^3$

i.e. cube root of 8000 = $(8000)^{1/3} = (20^3)^{1/3} = 20$ (answer).

Q. 1. B. Find the cube root of the following numbers.

729

Answer : First find factors of 729

$729 = 9 \times 9 \times 9$

For finding the cube root, we pair the prime factors in 3's.

$= 9^3$

i.e. cube root of 729 = $(729)^{1/3} = (9^3)^{1/3} = 9$ (answer).

Q. 1. C. Find the cube root of the following numbers.

343

Answer : First find the factor of 343

343 = 7 × 7 × 7

For finding the cube root, we pair the prime factors in 3's.

$= 7^3$

i.e. cube root of 343 = $(343)^{1/3} = (7^3)^{1/3}$ = 7 (answer).

Q. 1. D. Find the cube root of the following numbers.

-512

Answer : First find factors of - 512

-512 = (-8) × (-8) × (-8)

For finding the cube root, we pair the prime factors in 3's.

$= (-8)^3$

i.e. cube root of -512 = $(- 512)^{1/3} = (-8^3)^{1/3}$ = -8 (answer).

Q. 1. E. Find the cube root of the following numbers.

-2744

Answer : First find factors of -2744

-2744 = (-14) × (-14) × (-14)

For finding the cube root, we pair the prime factors in 3's.

$= (-14)^3$

i.e. cube root of -2744 = $(- 2744)^{1/3} = (-14^3)^{1/3}$ = -14 (answer).

Q. 1. F. Find the cube root of the following numbers.

32768

Answer : First find factor of 32768

32768 = 32 × 32 × 32

For finding the cube root, we pair the prime factors in 3's.

$= 32^3$

i.e. cube root of 32768 = $\sqrt[3]{32768} = (32^3)^{1/3} = 32$ (answer).

Q. 2. Simplify:

(1) $\sqrt[3]{\frac{27}{125}}$

(2) $\sqrt[3]{\frac{16}{54}}$

(3) If $\sqrt[3]{729} = 9$ **then** $\sqrt[3]{0.000729} = ?$

Answer :

(1) $\sqrt[3]{\frac{27}{125}}$

$\sqrt[3]{\frac{27}{125}} = \frac{\sqrt[3]{27}}{\sqrt[3]{125}} = \frac{\sqrt[3]{3\times3\times3}}{\sqrt[3]{5\times5\times5}} = \frac{\sqrt[3]{3^3}}{\sqrt[3]{5^3}} = \frac{3}{5}$ (answer).

(2) $\sqrt[3]{\frac{16}{54}}$

$\sqrt[3]{\frac{16}{54}} = \frac{\sqrt[3]{8}}{\sqrt[3]{27}} = \frac{\sqrt[3]{2\times2\times2}}{\sqrt[3]{3\times3\times3}} = \frac{\sqrt[3]{2^3}}{\sqrt[3]{3^3}} = \frac{2}{3}$ (answer).

3) If $\sqrt[3]{729} = 9$ then $\sqrt[3]{0.000729} = ?$

$$\sqrt[3]{0.000729} = \sqrt[3]{\frac{729}{1000000}} = \frac{\sqrt[3]{729}}{\sqrt[3]{100\times100\times100}} = \frac{\sqrt[3]{729}}{\sqrt[3]{100^3}}$$

We know that $\sqrt[3]{729} = 9$

So, $\sqrt[3]{0.000729} = \frac{9}{100} = 0.09$ (answer).